CONVERTING RIVER MURRAY MODELS FROM FINITE DIFFERENCE TO FINITE VOLUME

S. Suter[1], C. Mackay[1], D. McCullough[2], J. Kim[1], K. Yamagata[1]
[1]DHI Water and Environment, Sydney, NSW
[2]Department of Environment, Water and Natural Resources, Berri, SA

Assessing impacts of environmental strategies in wetlands and managing large flood events are integral elements in the field of floodplain and water resources management. The availability of hydraulic modelling tools greatly supports the decision making in such areas. In South Australia hydrodynamic models are used to support environmental water management in the River Murray channel and floodplain wetlands. The state's Department of Environment, Water and Natural Resources (DEWNR) has developed MIKE FLOOD hydrodynamic models of the River Murray and the Pike and Katarapko floodplains. These models were originally built as coupled 1D/2D fixed grid finite difference models, however they have been converted into coupled 1D/2D flexible mesh finite volume models. The flexible mesh models produce more detailed outputs since the mesh elements can vary in size allowing for higher spatial resolutions at areas of interest. In addition, these models can be run on GPU processors within reasonable run times while the same model built in classic grid run on CPU processors would be likely to exceed acceptable run durations. This paper examines the application of a coupled 1D/2D flexible mesh model to a complex river-floodplain system, which includes the River Murray and a number of smaller wetland branches and flood runners into the Pike and Katarapko floodplains. A coupled 1D/2D model with flexible mesh capability combines the advantages of both 1D and 2D models: channels with hydraulic structures including full control mechanisms can be simulated in 1D while the floodplain and major channels are represented in 2D with varying spatial resolutions. Flexible mesh elements can be aligned with floodplain features such as embankments and dikes. This paper highlights such benefits with respect to water resources planning and river operations. It also examines challenges of the fixed grid to flexible mesh conversion and the linking of 1D and 2D elements.

Keywords: 1D-2D coupling, flexible mesh, finite volume, GPU

Introduction

When modelling larger catchments (e.g. total number of cell sizes > 100,000) there is an increasing need for finite volume models with flexible mesh schematisation. This is based on the following:

1. With the varying cell sizes, areas of interest or areas where there are significant flow changes can be specifically resolved by smaller mesh elements while still being able to capture the full extent of the floodplain with larger mesh elements.

2. Elements in the flexible mesh scheme can be better aligned to channel and floodplain features (e.g. meandering river geometries) than the rectangular cells in the fixed grid scheme.

3. The finite volume method is more sophisticated in terms of handling complex and rapidly changing flow regime than the finite difference method (Engineers Australia, 2012).

4. The algorithm used in the finite volume code can be run on Graphical Processing Units (GPU) for the software used in this study (MIKE FLOOD FM) and run times significantly reduced.

With the availability of high-resolution topographic data and small mesh elements to represent rivers and channels in the two-dimensional (2D) model domain, there is often a reduced need for an additional one-dimensional (1D) model component. Furthermore, even though 1D modelling of the waterway is generally faster than a 2D model of the same waterway, solely 2D finite volume models have become an attractive alternative due to the shortened run times available with GPU processing capacity.

However, there are situations where the development of a 1D model linked to a 2D domain is the preferred option. This can occur in situations where there are narrow channels that could only be represented by very small mesh elements, which would greatly slow down the model runs. It can also be desirable to retain 1D elements where there are structures with complex hydraulics, or where structure control mechanisms need to be represented in the model. Often, such complexity is better implemented in a 1D model (e.g. MIKE 11).

Coupled 1D-2D flexible mesh models are the subject of this study. The Department of Environment, Water and Natural Resources (DEWNR) had originally built two hydraulic models of the River Murray and associated Pike and Katarapko floodplains in Southern Australia. The models were built as coupled 1D-2D models as there were numerous hydraulic structures with control mechanisms which needed to be represented in a 1D model component. Furthermore, there were several flood runners that were too narrow to be modelled in 2D with the adopted resolution. The 2D parts of the coupled models were developed as finite difference fixed grid models. In 2015 DEWNR decided to convert the finite difference fixed grid models to finite volume flexible mesh models. This was done to provide more detailed flood results by using smaller mesh elements in areas of interest, and using GPU parallelised processing to achieve shorter run times with these finer resolution models.

This particular project demonstrates the different aspects of current modelling practice in river-floodplain investigations and management. This paper highlights these aspects by looking at the application of the coupled 1D-2D flexible mesh model to the Pike and Katarapko floodplains.

Background

Finite difference vs finite volume

The dynamics of fluid flow is based on the principles of conservation of mass, momentum and energy. These dynamics are mathematically described by the Navier-Stokes equations (Hirsch, 1988). In case of surface water flooding application (where the wave length of free surface flow is much greater than water depth) the dynamics can be described by the Saint-Venant equations (the Shallow Water flow equations), which are depth-averaged forms of the Navier-Stokes equations (Popescu, 2014). These are described in a partial differential form and used for the schematisation of the numerical hydraulic model, which converts the conceptual model of the physical system into a numerical representation (Engineers Australia, 2012).

One of the considerations when applying the schematisation is the choice between the numerical representation techniques for the governing partial differential equations (PDEs). In the field of river and floodplain modelling there are two techniques which are generally selected from: finite difference or finite volume. It should be noted that the finite element technique is not described in this paper as most of the popular flood modelling software packages in Australia are based on either the finite difference or finite volume method. Further information about the finite element method can be found in Hughes (2000) and Zienkiewicz & Taylor (2000).

The finite difference technique solves the governing equations by replacing the derivatives with a finite difference approximation at the discrete nodes of the solution domain. The domain is discretised in fixed grids with the grid nodes being the locations where the values of the key variables are determined. At any other point in the domain that is not a discretisation node, the variables are obtained by interpolating between the known values at the nodes (Popescu, 2014). A slightly different approach is applied in the software package MIKE 21, where the discretised value is located at the centre of the grid cell and applied to the entire cell.

In the finite volume technique, the solution domain is subdivided into control volumes represented by volume-average values of the conserved variables of the Saint-Venant equations. The rate of change of observed variables is derived by integrating the cell-interface fluxes, which are conserved from one discretisation cell to another (Engineers Australia, 2012; Popescu, 2014). Various implicit and explicit integration methods can be used to form the discrete numerical approximations. Being based on the conservative integral form of the Saint-Venant equations, finite volume schemes are generally better able to robustly model both subcritical and supercritical flows, discontinuities such as hydraulic jumps and steep topographies (Engineers Australia, 2012).

Fixed grid vs flexible mesh

The fixed grid approach is generally based on a fixed rectangular grid with constant grid spacing. This approach uses a finite difference solution to the governing shallow water equations. The solution schemes for fixed grid models are numerically more efficient than flexible mesh models (for the same number of grid cells/mesh elements), with run times that are generally between four and eight times faster than the flexible mesh scheme (Engineers Australia, 2012).

The flexible mesh approach involves the discretisation of the model domain into elements of varying sizes. Hence, the spatial resolution can differ throughout the model domain, allowing the schematisation to concentrate computational effort on particular areas of interest. Flexible meshes can consist of triangular or quadrangular elements (or a combination of both) and provide great flexibility to represent complex geometries and boundaries in the study area (Engineers Australia, 2012; Suter et al., 2014; Mackay et al., 2015). Often, flexible mesh models can offset their reduced numerical efficiency by reducing the total number of elements compared to their fixed grid counterparts. This can be achieved by representing areas where less detail is needed using larger mesh elements, and thereby reduce the total number of computational elements.

A visual comparison between flexible mesh and fixed grid models is shown in Figure 1.

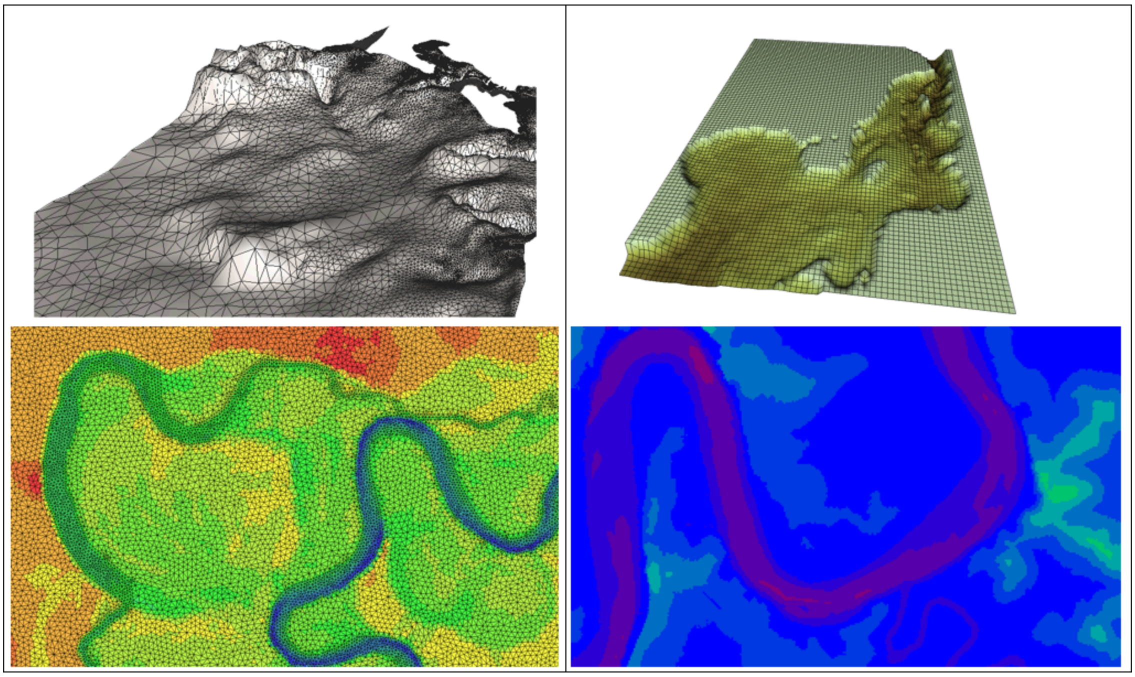

Figure 1 Flexible mesh models (left) and fixed grid models (right)

GPU

Graphical Processing Units (GPUs) are now routinely adopted in mainstream scientific and engineering computing. They can be purchased at relatively low cost and have become a viable alternative to using Computational Processing Units (CPUs) for numerical calculations, and have been widely adopted by water resources modellers. Several modelling codes (e.g. MIKE 21 Flexible Mesh) have been rewritten to benefit from the parallelised and distributed architecture of the GPU. This leads to significant reduction in run times because the algorithm describing the PDEs can be solved simultaneously on different cores on the run machine, with each core working on its own set of data.

Model application

Model software

MIKE FLOOD Flexible Mesh (MIKE FLOOD FM) was used for the River Murray models in the Pike and Katarapko floodplain project. MIKE FLOOD FM allows for dynamic linking of 1D and 2D domains.

The 1D domain is represented in MIKE 11 and its hydrodynamic module was used to simulate river flows and water levels. This module solves the vertically integrated Saint-Venant equations (MIKE by DHI, 2014a).

The 2D domain is represented by the finite volume based flexible mesh modelling code MIKE 21 FM. It is based on the numerical solution of the 2D incompressible Reynolds averaged Navier-Stokes equations, assuming hydrostatic pressure. The PDEs are discretised into non-overlapping mesh elements, which can be either triangular or quadrilateral (MIKE by DHI, 2014b). The finite volume method used in MIKE 21 FM determines the fluxes between the mesh elements by solving an Equivalent Riemann Problem. The approach treats the problem as 1D in the direction perpendicular to each element interface (Guinot, 2003).

Study area

The Murray River in the study area is characterised by a highly meandering main channel that flows within a wider but generally constrained floodplain. The main river channel varies in width from as little as 60 m to over 200 m.

The weirs along the river create a series of permanent water bodies on the floodplains in each weir pool which are connected to the main flow channel by one or more low level channels. During flood events the floodplains become inundated through higher level flood runners and ultimately through large scale overbank inundation processes.

Model overview

The existing MIKE FLOOD models of Pike and Katarapko each combined a MIKE 11 and a MIKE 21 finite difference (classic) grid model. In the upgraded MIKE FLOOD models, the MIKE 21 models were converted to flexible mesh models. The domains of the 2D models were extended further upstream and downstream relative to the existing MIKE FLOOD model domains.

While the main model upgrade involved the MIKE 21 conversion from finite difference grid to flexible mesh, there were some additional modifications of specific study features. Furthermore, the MIKE 11 model was modified at certain locations to better match the local settings. However, since the focus of this paper is on the flexible mesh generation and 1D-2D linking, these modifications are not described further in this paper.

As a general overview, only narrow flood runners and watercourse sections including hydraulic structures were represented in 1D while the entire floodplain and most of the waterways (River Murray and Katarapko Creek) and anabranches were represented in the 2D domain.

LiDAR data (2 m horizontal resolution), roughness data, location of blocking banks and evaporation data were provided by DEWNR.

Both the existing and upgraded model domains and river networks are shown in Figure 2 (Pike) and Figure 3 (Katarapko).

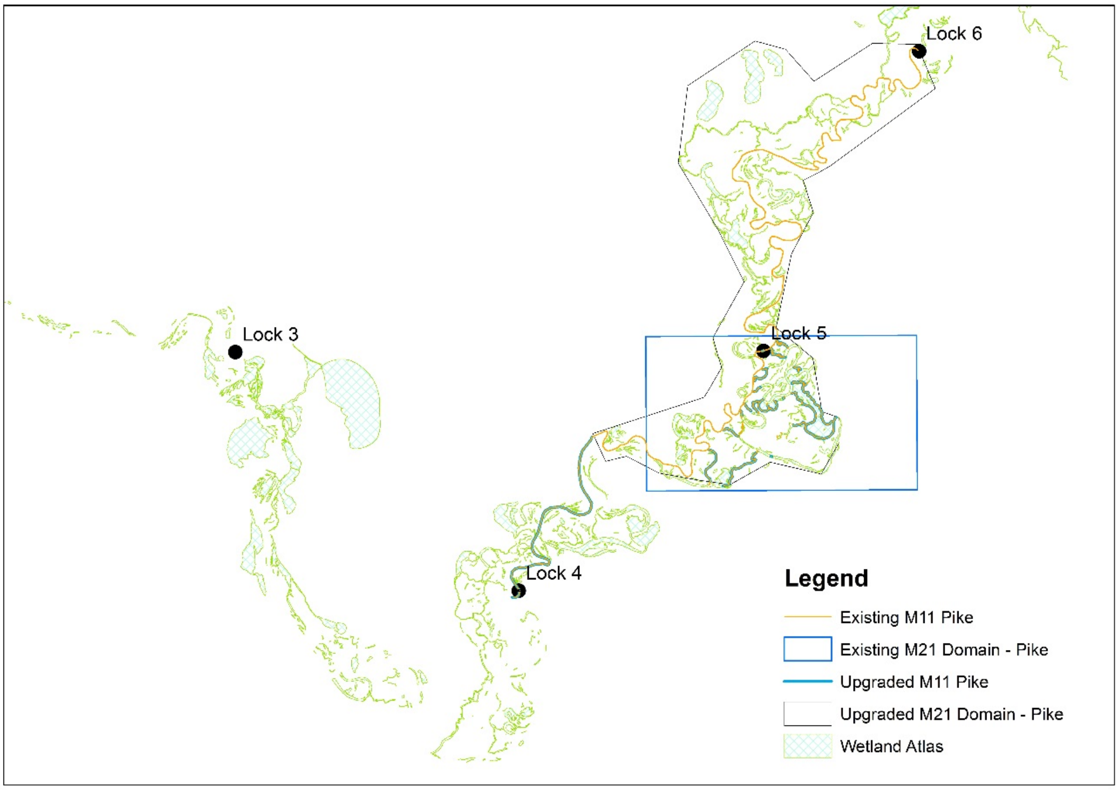

Figure 2 Existing and upgraded Pike model

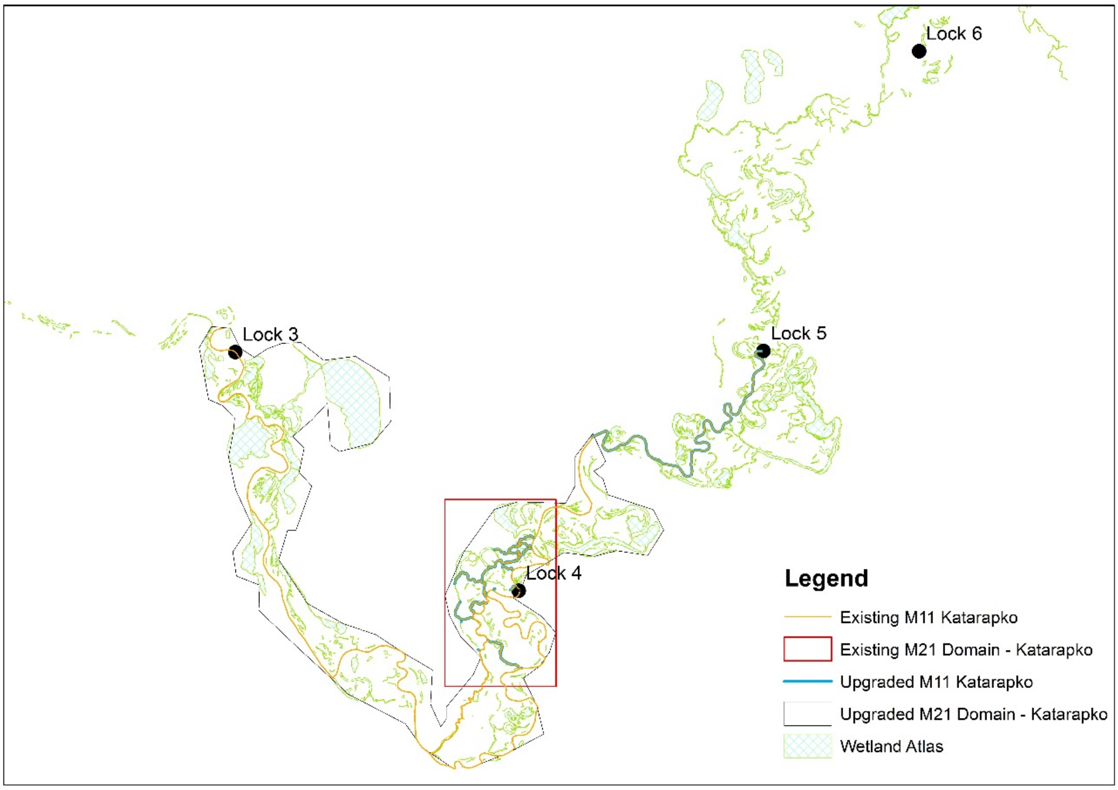

Figure 3 Existing and upgraded Katarapko model

Generation of the flexible mesh

In order to implement topographic data within the model and to represent the locations of the boundary conditions, MIKE 21 FM requires a mesh file. The topography data is interpolated over the discretised elements which the mesh is made of. The generation of a mesh is likely to be the most time-intensive process of the entire model setup. As it is the primary input needed for the development of a hydraulic model, it needs to be well planned and constructed with care. Representation of flood pattern and overland flowpaths and storages relies heavily on an adequate topography definition of the study area (Engineers Australia, 2012), and hence on the mesh schematisation. If certain drainage features in the study area that are likely to influence the passage or storage of flows are not represented in the mesh, the modelling results will be negatively affected. For example, small channels can play important roles in floodplain inundation and drainage. Inappropriate mesh alignment or too few mesh elements across the channel and banks should be avoided in order to achieve realistic flow conveyance.

As there are more aspects to include in the flexible mesh approach, its generation is more time consuming than building a fixed grid. Nonetheless, the setup of a fixed grid model still requires care. The grid size has to be small enough to model the hydraulic effects of flow splits, sharp bends and narrow channels running diagonal to the grid alignment. However, if the grid size is solely chosen to represent such features, it will lead to a computationally slow model that produces unnecessarily detailed results for the wider floodplain (Mackay et al., 2015).

In this study the mesh was generated using the built-in mesh generator tool for MIKE 21 FM. The tool allows the user to create polygons that define the model extent and regions with local properties such as different mesh element sizes or shapes (triangular or quadrangular). Furthermore, the tool creates an interpolated topographic surface based on scatter points extracted from the topographic raster file.

Triangular elements were used in the models for the entire domain. These were aligned to the watercourses that were represented in 2D, such as main channel and some smaller anabranches. Smaller elements were used to represent these watercourses, generally allowing for more than 3 elements across the channel. This ensured the channels were represented with a high enough spatial resolution and therefore including channel bed and bank elevations in the final topographic surface.

The area represented in MIKE 11 was excluded from the mesh. This area is typically defined along the 1D branches with the width changing according to the cross-section width where conveyance is allowed to occur.

Blocking banks had to be implemented into the model. These are constructed to protect a particular area from flooding and to control flow into the wetland through specific inlets. The blocking banks were represented in the mesh by aligning the elements to the imported shape file of the bank alignment. So called arcs were set along these lines to ensure the elevation of the top of the blocking bank would be extracted during the topographic surface generation.

Once the mesh elements were created, the raster file was used to interpolate a topographic surface over the generated mesh. A zoom-in area of the flexible mesh in the Pike model with interpolated topography is shown in Figure 4. Smaller mesh triangles are used in the River Murray and larger ones in the floodplain. The figure also highlights the mesh exclusion zones for two flood runners and a gate structure on the River Murray (grey areas). These are modelled in MIKE 11 and linked to the 2D domain.

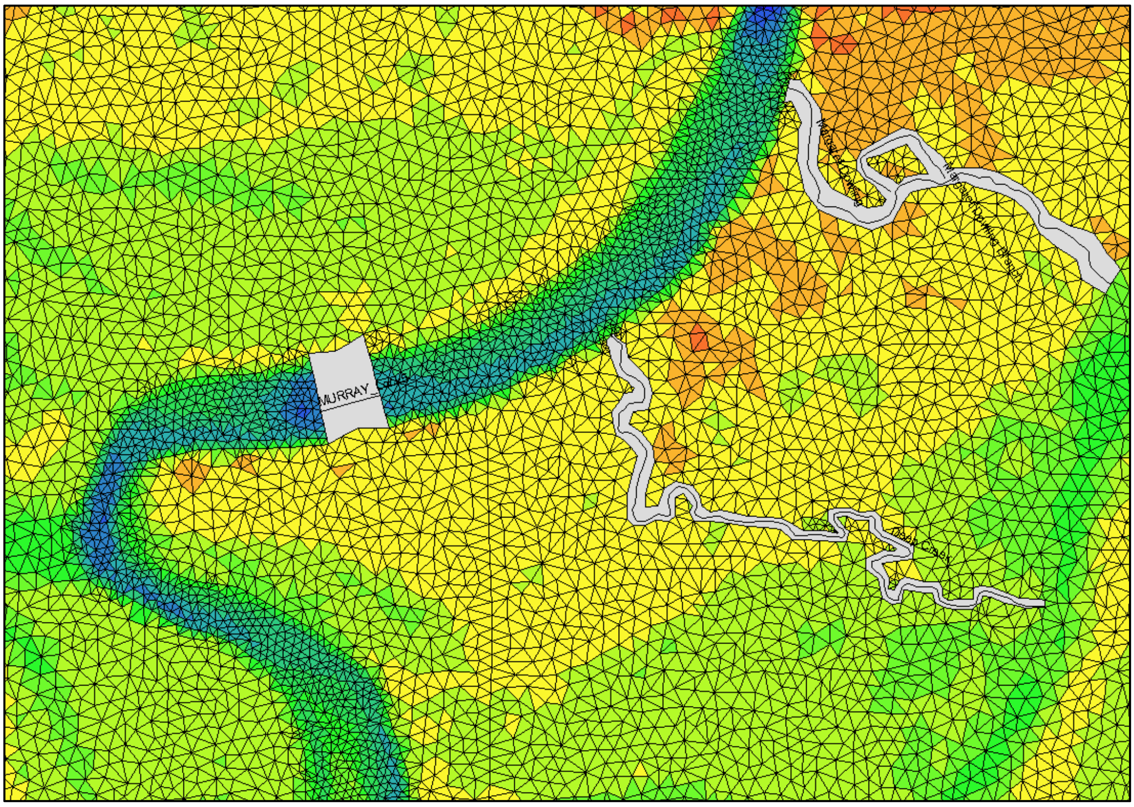

Figure 4 Generated mesh including 1D exclusion zones

Boundary and initial conditions

The boundary conditions were set in the MIKE 21 FM models at the upstream and downstream ends of the mesh. The open boundaries of the 1D model are linked to the 2D domain and the 1D conditions are replaced by the 1D/2D couple.

An upstream discharge boundary was imposed at the location where the River Murray enters the model domain.

The downstream end was artificially extended and a fixed water level boundary was imposed across the channel and floodplain. In the extended area a smooth steep dummy slope was assigned allowing water to flow freely out of the model domain. This approach was chosen to prevent backwater effects from affecting results in the area of interest.

1D-2D coupling

MIKE FLOOD combines the 1D and 2D models by introducing different types of links that allow for information transfer between the two models. In this study, standard links and lateral links were used.

Standard links were set at locations where one or more mesh elements were to be linked to the start or end of a MIKE 11 branch. As mentioned above, the standard links act as boundary conditions and replace the open end boundaries set in MIKE 11. Manual adjustment of the cross-sections in the MIKE 11 model was carried out at the standard link locations to match the 2D topography. This was done to improve the accuracy of flow conveyance representation into and out of the channels.

A momentum factor of 0.5 was generally used in the standard links parameter setup (1.0 for River Murray). This parameter controls whether momentum exchange is included with 1 and 0 being full momentum exchange and no momentum exchange, respectively. Reducing momentum exchange generally has a stabilising effect on the model run. It should be noted however, that reducing momentum transfer might not give an accurate solution (e.g. cases where momentum conservation is critical).

Lateral links were created to represent the overtopping flow from the MIKE 11 branches. Lateral links act like levees and use a modified form of the weir equation to define flow transfer across the lateral links. The lateral links were set along the mesh lines bordering the MIKE 11 exclusion zones.

The greatest height of the two model components was mostly used for the levee height, which is used in the weir equation defining flow transfer across the links. For example, if the cross-section elevation in MIKE 11 at the lateral link location was larger than the elevation in the mesh element for the same location, the MIKE 11 elevation would be used for the levee height.

Hydraulic structures

As mentioned above, hydraulic structures were implemented in the MIKE 11 model. This was done as MIKE 11 offers better representation of complex structures, and allows for time-varying or internal variable dependent control. Control structures can be used whenever flow through a structure needs to be regulated by the operation of a movable gate or when there is a need to control flow by other mechanisms (MIKE by DHI, 2014d). For example, a discharge type control structure can be implemented for cases where a specific discharge is to be forced through the structure depending on a defined water level difference across the structure.

Types of hydraulic structures included in the model were weirs, culverts and control structures. These were assigned to a short 1D branch, which was linked to the MIKE 21 FM model by a standard link.

Flood and dry

When flood water propagates through the model, mesh elements that were initially dry have to become wet, which means they have to be integrated into the computation. On the other hand, once the flood levels recede, they have to be taken out of the computation. Different 2D flood modelling software packages have different wetting and drying algorithms (Engineers Australia, 2012). In the MIKE 21 FM model used in this study, the advanced flooding and drying mode setting was applied. Depending on the local modelled water depth in a mesh element, this mode chooses one of the following options:

- mesh elements are fully included in the calculations and both momentum and mass fluxes are calculated,

- mesh elements are included in the calculations, but momentum fluxes are set to zero while calculating the mass fluxes,

- mesh elements are removed from the calculation.

Run settings

A “low order, fast algorithm” solution technique with adaptive time steps was used for the shallow water equations. MIKE 21 FM uses an adaptive timestep that is adjusted during the simulation to fulfil the CFL criterion required to ensure numerical scheme stability. Furthermore, the run time step was constrained to be between minimum and maximum values of 0.5 seconds and 5 seconds.

Model results

Both Pike and Katarapko models were run for natural flooding and managed flooding scenarios, and were validated by comparing the modelled flood extent to an aerial imagery for a specific event. An example of the validation is shown in Figure 5, which compares an aerial image of the 2011 flood against the simulated water depth for the natural flooding scenario in the Pike floodplain. Overall, the modelled extent agrees well with the flood image.

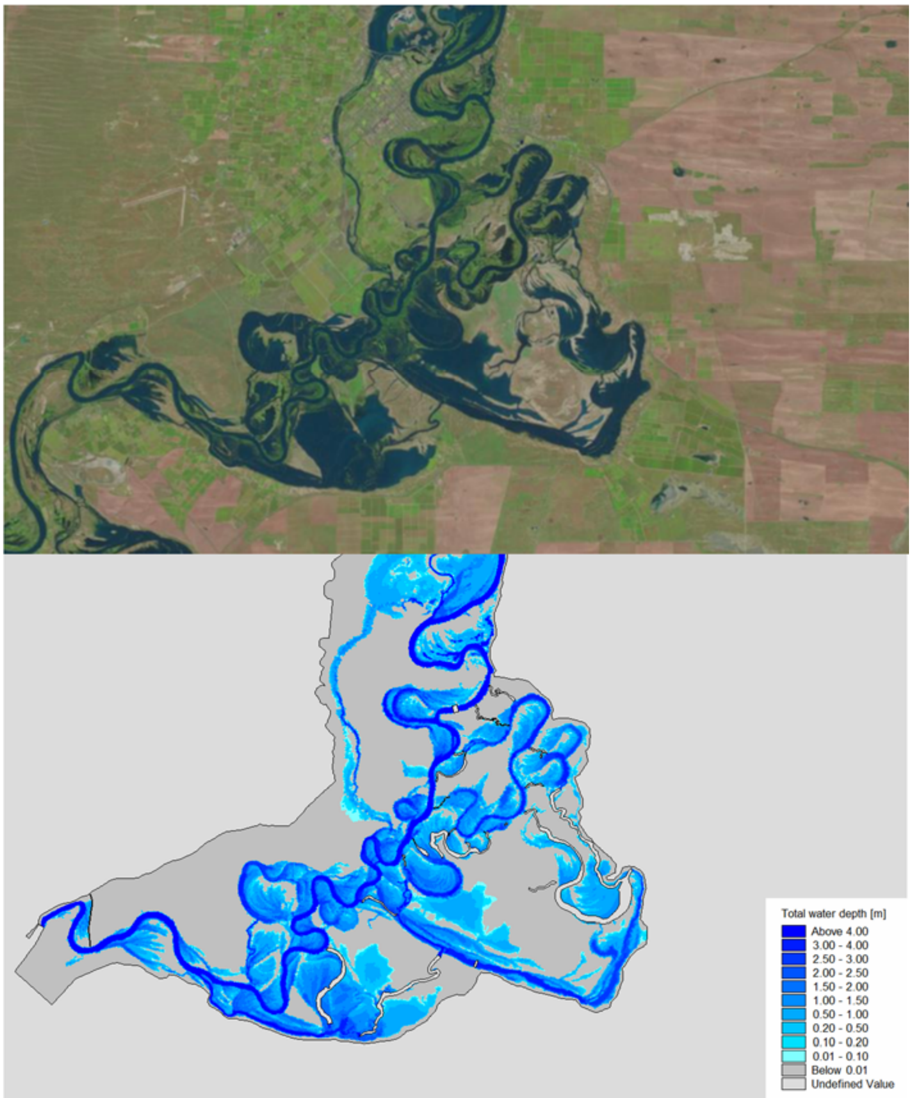

Figure 5 Extent of natural flooding for the Pike model – Aerial image (top) and modelled water depth (bottom)

It is of interest to compare the level of detail and structure in the two models. Figure 6 shows an area in the Pike model where a smaller anabranch joins up with the River Murray. The figure displays higher and lower topographic elevations in red and blue, respectively. For the anabranch, the flexible mesh model has a higher number of elements within the channel and representing the channel bank. The smaller element sizes and increased number of calculation points produces a better representation of the channel conveyance and storage, and higher resolution result outputs.

It should be noted that the River Murray was modelled in 1D in the original fixed grid model, while being represented in 2D in the converted flexible mesh model. It was therefore not possible to directly compare water level, flow and velocity results. The channel topography in the fixed grid model of Figure 6 is superimposed onto the grid for display reasons only. In the original model the channel is excluded from grid.

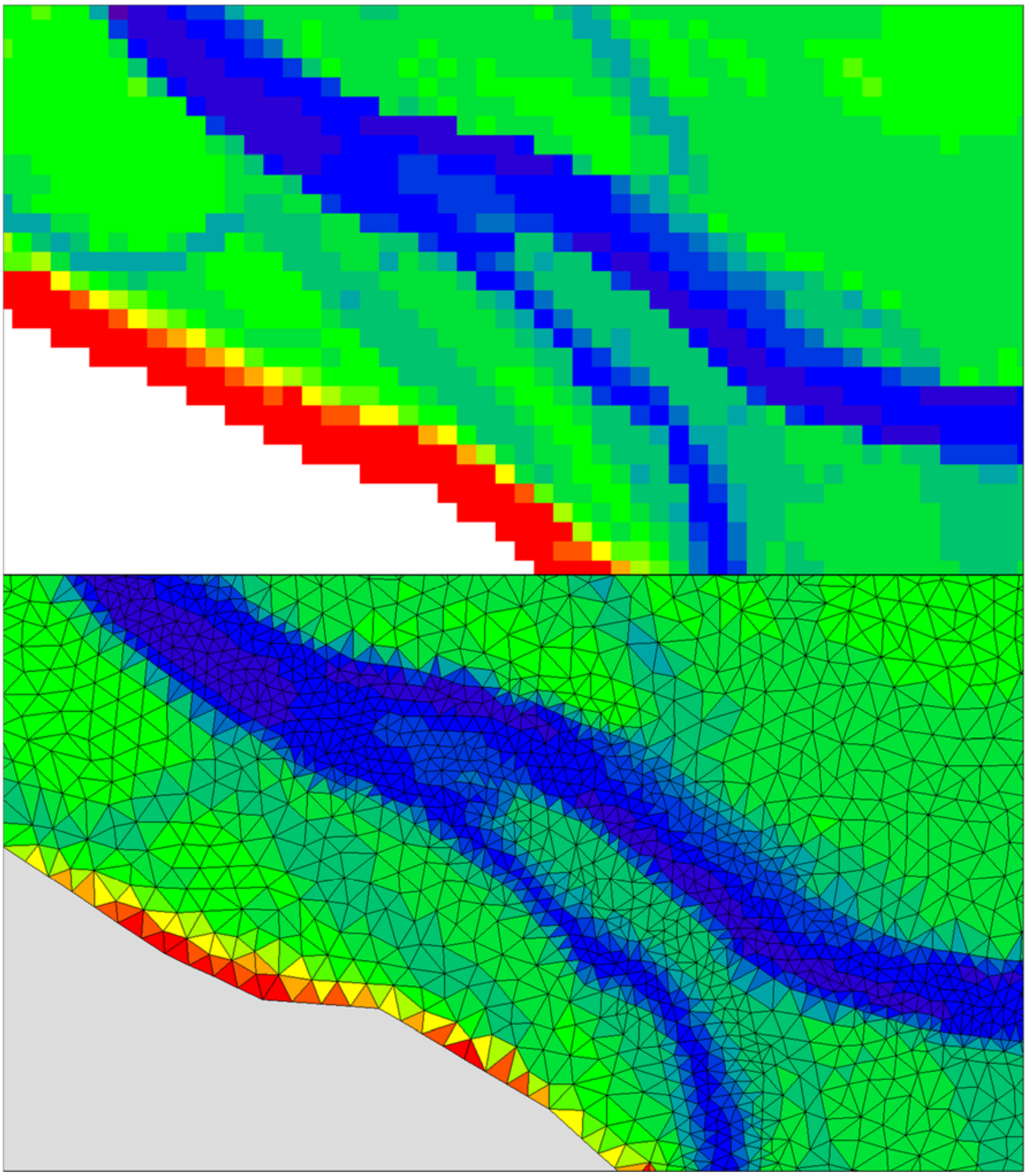

Figure 6 Fixed grid model, 30 m spacing (top) and flexible mesh model (bottom)

Conclusions

DEWNR uses detailed hydrodynamic modelling to assess the impacts of water management strategies in the Pike and Katarapko floodplains. Modelling outputs that cover a range of flows are becoming more important for a range of stakeholders for planning and river operations. It is therefore essential to have hydrodynamic models that are able to represent a large study area with accurate representation of complex features. Such models also need to produce enough detail while running within reasonable run times.

This study highlights that such objectives can be achieved with the development of a coupled 1D-2D flexible mesh model running on a GPU processor. The 2D flexible mesh scheme provides the ability to vary the element size within the modelling area to balance computational speed against accuracy. The 1D model is effective at modelling flows through complex hydraulic structures, particularly those using control mechanisms. 1D/2D flexible mesh modelling provides a means of combining these advantages within a single model. However, it is emphasised that generating a suitable 2D mesh and accurate 1D-2D linkages requires care and an understanding of the underlying model behaviour.

Acknowledgments

We would like to thank DEWNR for supporting this paper through provision of LiDAR data, fixed grid model files and aerial imagery.

'SARFIIP is a $155 million investment program funded by the Australian Government and implemented by the South Australian Government to improve the watering and management of River Murray floodplains in South Australia's Riverland'.

References

Engineers Australia, 2012. Australian Rainfall and Runoff – Revision Project 15: Two-dimensional Modelling in Urban and Rural Floodplains. Engineers Australia (Water Engineering), Barton.

Guinot, V. 2003. *Godunov-type Schemes: An Introduction for Engineers*. Elsevier.

Hirsch, C. 1988. *Numerical Computation of Internal and External Flows, Volume 1: Fundamental of Numerical Discretization*. Wiley and Son, Chichester.

Hughes, T. 2000. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis.* Dover Publishers.

Mackay, C., Suter, S., Albert, N., Morton, S., and Yamagata, K. 2015. Large scale flexible mesh 2D modelling of the Lower Namoi Valley. Conference Paper. FMA 2015.

MIKE by DHI. 2014a. *MIKE 11 - A Modelling System for Rivers and Channels – Reference Manual*. DHI Water and Environment.

MIKE by DHI. 2014b. *MIKE 21 & MIKE 3 Flow Model FM Hydrodynamic and Transport Module – Scientific Documentation.* DHI Water and Environment.

MIKE by DHI. 2014c. *MIKE FLOOD User Manual.* DHI Water and Environment.

MIKE by DHI. 2014d. *MIKE 11 User Manual.* DHI Water and Environment.

Popescu, I. 2014. *Computational Hydraulics – Numerical methods and modelling*. IWA Publishing, London.

Suter, S., Singh, G., and Britton, M. 2014. *Changes in dam break hydrodynamic modelling practice*. Conference Paper. ANCOLD 2014.

Zienkiewicz, O., and Taylor, R. *The Finite Element Method: The Basis, Vol. 1.* Butterworth and Heinemann.